



Complex spectrum analyzer measurement principle

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I. Description of the dynamic chirp :

I. 1. Definition of the « chirp »

Let us consider an optical wave, whose electromagnetic field can be expressed in the complex form as :

$$E = A \exp[i2\pi f_0 t] = |A| \exp[i2\pi f_0 t + \varphi]$$

where A is the complex envelop, $i = \sqrt{-1}$, t is the time, f_0 is the frequency and φ is the phase of the optical wave.

The « chirp » designates the transient deviation of the instantaneous frequency f from the value f_0 , caused by a temporal variation of the phase φ .

The instantaneous frequency is given by :

$$f(t) = f_0 + \frac{1}{2\pi} \frac{d\varphi}{dt}$$

and the chirp is :

$$C(t) = f(t) - f_0 = \frac{1}{2\pi} \frac{d\varphi}{dt}$$

I. 2. Chirp in optical telecommunications :

In optical telecommunications, the light is used as the information carrier, and the data bits are encoded by a modulation of the optical intensity. The bit 1 is represented by the maximal intensity value I_{max} and the bit 0 by the minimal intensity value I_{min} .

The data bit series is generated by varying the intensity of a laser source with a modulating signal $M(t)$, either directly or by using an external modulator, and by transmitting the resulting signal to a fiber. This signal is received by a detector at the end of the fiber and the information is retrieved by measuring the temporal intensity modulation.

The most common bit rate is 9.95328 Gb/s (standard OC192/STM64), but systems working at 39.81312 Gb/s (OC768/STM256) are now widely developed and used. Two transmission formats are used : NRZ (for transmitting a bit 1, the intensity stays at I_{max} during the bit period) and RZ (the intensity stays at I_{max} during the first half of the period and goes to I_{min} during the second half).

The temporal profile of the signal intensity is thus a succession of transitions between I_{max} and I_{min} (see figure 1). Generally, the modulation of the light does not affect only the amplitude of the envelop, but the phase too, because of the coupling between amplitude and phase induced by direct and external modulation processes. The resulting dynamic chirp presents generally strong peaks at rising and falling edges of the intensity. Its maximal order of magnitude is of several GHz, whereas the carrier frequency f_0 is about 194 THz (corresponding to the wavelength $1.55\mu\text{m}$ used in fiber optic communication systems).

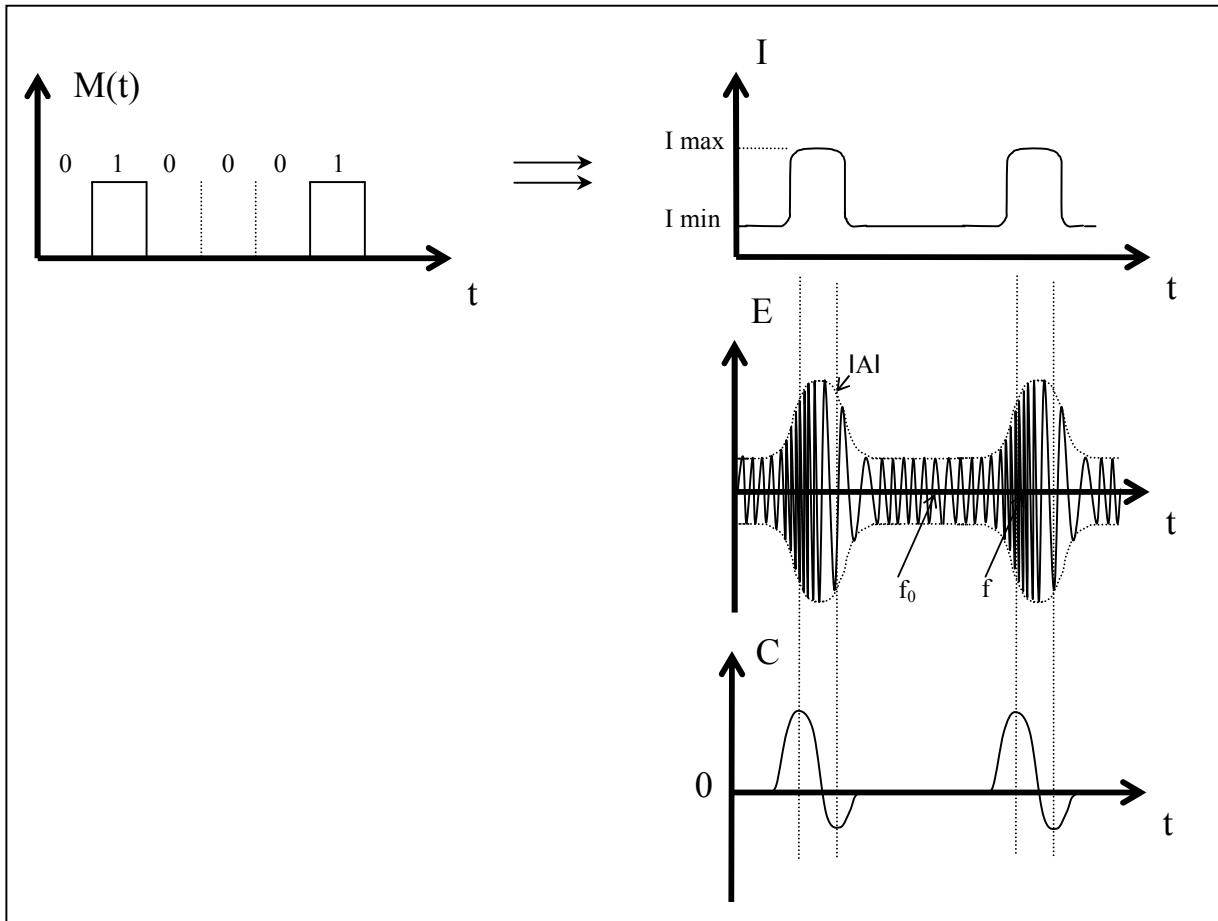


Figure 1 : illustration of the dynamic chirp during optical bit generation

II. Chirp measurement : method implemented in APEX AP2440A series

The method implemented in APEX AP2440A series has been developed by France Telecom researchers¹ and is patented by France Telecom. It is based upon a spectral analysis of the optical field, and permits to extract the amplitude and the phase of each frequency component (whereas classic spectrum analyzers measure the power spectral density, giving only the amplitude). Knowing the amplitude and the phase of each spectral component, the temporal variations of the amplitude and the phase can be calculated by Fourier transform, giving the intensity and the chirp as a function of time.

This method is adapted to periodic modulation signals only. Let T be the period $F = 1/T$ the fundamental frequency of the modulation signal $M(t)$. Then the complex envelop of the optical field has the same period and thus the Fourier expansion of the complex field can be written as :

$$E(t) = A(t) \exp[i2\pi f_0 t] = \sum_k A_k \exp\{i[2\pi(f_0 + kF)t + \Phi_k]\}$$

The corresponding spectrum is composed of discrete and regularly spaced lines (see figure 2) : one is located at the carrier frequency f_0 , surrounded by modulation lines located at $f_0 + kF$. A_k is the amplitude and Φ_k is the phase of the line k .

The amplitude values are given by the power levels of the spectral lines. The measurement of the phase values is done by means of the following method.

A sinusoidal modulation at the frequency $F/2$, synchronized with the modulation signal $M(t)$ and with a variable delay τ , is applied to $E(t)$. Each line of the original spectrum gives birth in the resulting spectrum to two symmetrical lines located at $\pm F/2$ from this line. The line k generates two lines $L_+(k)$ and $L_-(k)$ at $(f_0 + kF \pm F/2)$, and the line $(k + 1)$ generates two lines $L_+(k + 1)$ and $L_-(k + 1)$ at $[f_0 + (k + 1)F \pm F/2]$ (see figure 2).

As $L_+(k)$ and $L_-(k + 1)$ have the same frequency, they interfere and the resulting power is proportional to $\cos[2\pi F\tau + \Phi_{k+1} - \Phi_k]$. The phase difference $\Phi_{k+1} - \Phi_k$ is retrieved by measuring the power variations as a function of τ . By this way, all the phase values Φ_k can be calculated gradually, starting from $k = 0$.

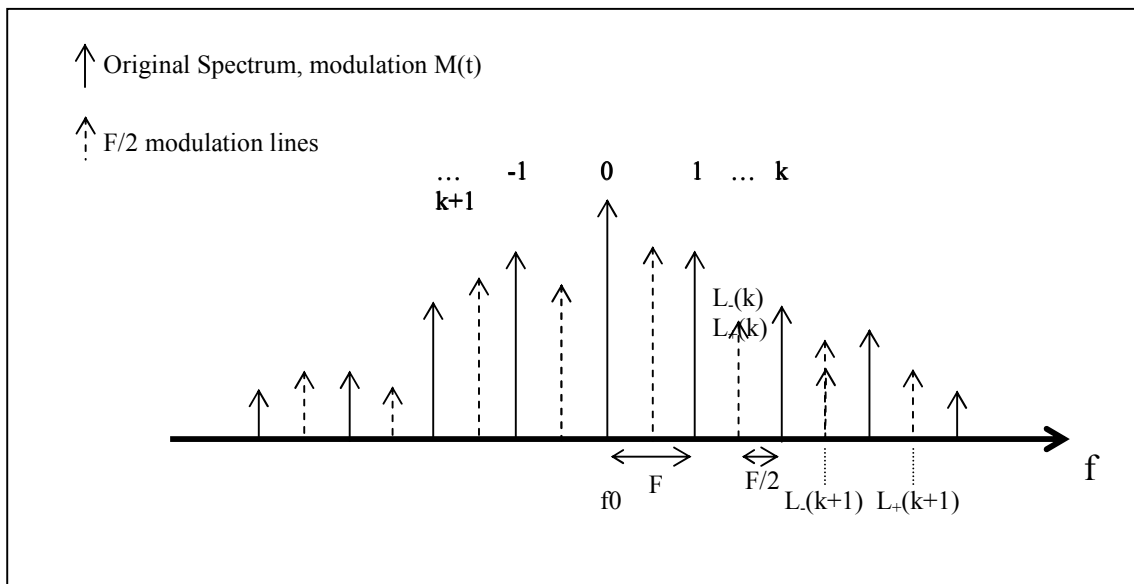


Figure 2 : discrete spectrum corresponding to a periodic modulation $M(t)$ ($F =$ fundamental frequency, $f_0 =$ carrier frequency) and effect of a sinusoidal modulation at the frequency $F/2$.

Knowing A_k and Φ_k , an FFT algorithm calculates

$$A(t) = \sum_k A_k \exp\{i[2\pi kFt + \Phi_k]\} = |A(t)| \exp[i\phi(t)]$$

which leads to the temporal profile of the intensity (the shape of the optical signal) :

$$I(t) = |A(t)|^2 / 2$$

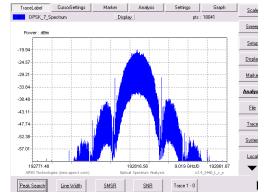
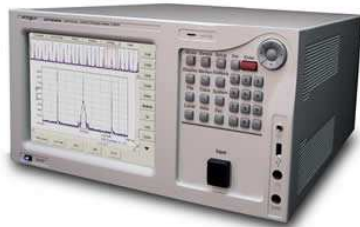
and the temporal profile of the chirp

$$C(t) = \frac{1}{2\pi} \frac{d\phi}{dt}$$

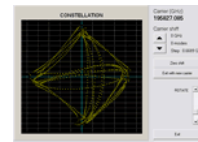
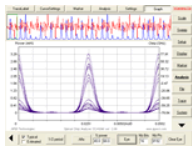
These profiles are calculated over a time window equal to T and with a time resolution $\Delta t = T/N$, where N is the number of lines analyzed in the original spectrum.

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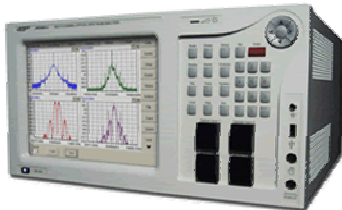
AP2040A series Ultra-high resolution Optical Spectrum Analyzer



AP2440A series Complex Spectrum Analyzer



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